**Assignment: Estimating Mean Durability of Print-Heads Using Confidence Intervals**

**Background**

In quality control, especially for high-value items such as print-heads, destructive testing is used to measure durability. Because testing destroys the item, only a small sample size can be used. This assignment focuses on constructing 99% confidence intervals for the mean durability of print-heads (in millions of characters printed before failure) using both the sample standard deviation and a known population standard deviation.

**Scenario**

A manufacturer randomly selected 15 print-heads and tested them until failure. The durability values (in millions of characters) are:

1.13, 1.55, 1.43, 0.92, 1.25, 1.36, 1.32, 0.85, 1.07, 1.48, 1.20, 1.33, 1.18, 1.22, 1.29

**a. 99% Confidence Interval Using Sample Standard Deviation**

**Step 1: Calculate the Sample Mean and Sample Standard Deviation**

Let;

:

 n=15 (sample size)

 xˉˉ = sample mean

 s = sample standard deviation

xˉ= n/∑xi =15/ 18.66​ = 1.244

s=∑(xi​−xˉ)2​​/n−1 ≈ 0.192s

**Explanation**

**The t-distribution is used instead of the normal distribution because:**

* **The sample size is small (n<30n < 30n<30).**
* **The population standard deviation is unknown.**
* **We assume the data is approximately normally distributed.**

**Step 2: Use the t-distribution**

Because the sample size is small and the population standard deviation is unknown, we use the **t-distribution**.

Degrees of freedom df=n−1=14

**Step 3: Construct the Confidence Interval**

**Final Result**

CI=xˉ±tα/2,n−1​⋅n​s

​CI=1.244±2.977⋅15​0.192

​CI=1.244±2.977⋅0.0496

CI=1.244±0.1478

**b. 99% Confidence Interval Using Known Population Standard Deviation**

Now, assume the **population standard deviation σ=0.2\sigma = 0.2σ=0.2** million characters is known.

**Step 1: Use the z-distribution**

Since σ\sigmaσ is known, use the **z-distribution** instead of the t-distribution.

From the **z-table**, the **critical value z0.005=2.576z\_{0.005} = 2.576z0.005​=2.576**

**Step 2: Construct the Confidence Interval**

**Final Result**

CI=xˉ±zα/2​⋅n​σ​

CI=1.244±2.576⋅15​0.2

​CI=1.244±2.576⋅0.0516

CI=1.244±0.133

99% Confidence Interval: (1.111, 1.377)​

**Explanation**

The **z-distribution** is used here because:

* The population standard deviation is **known**.
* The central limit theorem justifies using z even for moderate sample sizes.

**Conclusion**

The manufacturer can be 99% confident that:

* The true mean durability lies between **1.096 and 1.392 million characters** using sample statistics.
* If the population standard deviation is known, the interval tightens to **1.111 to 1.377 million characters**.

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